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NUMERICAL SOLUTIONS OF POLYNOMIAL EQUATIONS

by

Tadeusz Leser
Henry Wisniewski

August 1968

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TECHNICAL NOTE NO. 1699

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NUMERICAL SOLUTIONS OF POLYNOMIAL EQUATIONS

ABSTRACT

This report describes the FORTRAN Subroutine POLYR and a related complete program BRL-RSSR for finding all roots (real and complex) of a real polynomial equation

$$P(x) = \sum_{i=0}^N A_i x^{N-i} = 0.$$

The method which is used combines the root squaring and the subresultant (extracting quadratic factors) procedures for calculating all roots, even multiple roots, of real polynomials. Reconstruction of the coefficients from the product

$$\prod_{i=1}^N (x - \text{Root}_i),$$

which is included, serves as a means of checking.

The described subroutine and program were adapted from a similar program in double precision described in Report ANL 6987, Argonne National Laboratory by Erwin H. Bareiss and Ronald Hamelink.

ROOTS OF POLYNOMIALS

Subroutine POLYR and BRL-RSSR Program

General Description. A FORTRAN subroutine and a related complete program for finding all roots (real and complex) of a real polynomial equation

$$P(x) = \sum_{i=0}^N A_i x^{N-i} = 0$$

are available for use on the BRLESC computer.

Method. The root squaring and subresultant (extracting quadratic factors) procedure "RSSR" for finding real, complex and repeated roots of real polynomials and checking them by reconstructing the coefficients from the product

$$\prod_{i=1}^N (x - \text{ROOT}_i)$$

is described in detail in

Report ANL 6987

Argonne National Laboratory

by Edwin H. Bareiss and Ronald Hamelink .

Authors. The routine under the name RSSR was written in FORTRAN by the above authors for the IBM-704 and CDC-3600 computers using double precision arithmetic. The modifications and additions necessary to make it run on BRLESC in single precision were performed by Henry Wisniewski of the Computing Laboratory of BRL.

Modifications to the Subroutine POLYR. When compared with the original FORTRAN ANL subroutine (pp 49-73) the BRL subroutine POLYR differs as follows:

a) The Main Program, which does input and output and calls RSSR, has been entirely replaced (see tabulation).

b) Subroutine RSSR

No changes.

c) Subroutine ROOTSQ

Changes:

LINE	STATEMENT
1+6	IF (KM) 11, 4, 11
11	DO 4 L=1, K M

d) Subroutine REALROOT

Changes:

LINE	STATEMENT
11+1	T=2**M*I

Reason: Mixed type arithmetic is not allowed in BRLESC

FORTRAN.

e) Subroutine COMPRCOT

No changes.

f) Subroutine TEST

Changes:

LINE	STATEMENT
11	IF (IB(3) - 74) 100, 100, 12
100	IF (IB(3) + 74) 12, 101, 101
18	IF (IB(2) - 74) 102, 102, 12
103	IF (IB(2) + 74) 12, 103, 103
103+1	IF (IB(3) - 74) 104, 104, 12
104	IF (IB(3) + 74) 12, 105, 105

g, h, i) Subroutines SUBRES, RECON, QUADRIV

No changes.

j) Subroutine DOUBLOG

Changes:

LINE	STATEMENT
1	PRINT 2
3	To = Ix
3+1	y = ALOG(x) + To*ALOG(T)

Reason: Mixed type arithmetic is not allowed.

k) Subroutine DOUBLEXP

Changes:

LINE	STATEMENT
First	Z = EXP(X) \$IZ = 0

l) Subroutine ADD

Changes:

LINE	STATEMENT
10	IF (IDIFF) 11, 12, 11
12	DO 11 I = 1, IDIFF

m,n) Subroutines SUBTRACT and SCALE

No changes.

o) Subroutine UNSCALE

Changes:

LINE	STATEMENT
First + 1	IF(IX + 84) 3, 4, 4
4	IF(IX - 84) 5, 5, 6

Reason: Maximum size of IX on BRLESC is 84

6	X = 1.0E + 153
---	----------------

Reason: Maximum number on BRLESC is 1.34078E154

6+2	PRINT 7
7	FORMAT (25H0EXP. OVERFLOW IN UNSCALE)

Operating Instructions for the Subroutine POLYR. They are replaced by:

Entry is made by the statement

CALL POLYR (N, COEFF, ROOTS, D)

where

N = Degree of the polynomial. (A fixed point integer.)

COEFF = One-dimensional array of length (N+1) which contains the locations of the coefficients.

ROOTS = Two-dimensional array of length (2xN) which contains the roots.

D = One-dimensional array of length (N+1) which contains the reconstructed polynomial from the roots and the first coefficients.

The coefficient of the highest degree is stored first, followed by the remaining coefficients in decreasing order of degree.

Storage

COEFF (1)	A_0
COEFF (2)	A_1
COEFF (3)	A_2
...	...
COEFF (N+1)	A_N

The results are:

ROOTS (1, 1)	Real Part	ROOTS (2, 1)	Imaginary Part
ROOTS (1, 2)	Real Part	ROOTS (2, 2)	Imaginary Part
ROOTS (1, 3)	Real Part	ROOTS (2, 3)	Imaginary Part
...
ROOTS (1, N)	Real Part	ROOTS (2, N)	Imaginary Part

The coefficients of the reconstructed polynomial.

D(1)	A_0
D(2)	A_1
D(3)	A_2
...	...
D(N+1)	A_N

This subroutine is available on cards only. (See Appendix A)

Modifications to the Program RSSR. The BRL-RSSR Program contains the same modifications as POLYR with the exception of a Main Program which is retained as shown in Report ANL 6987 with the following changes:

LINE	STATEMENT
2+1 and 3	Cancelled
4	PRINT 2, TITLE
9+3	Cancelled
10+1	"
14+1 through 18	"
19+2	"
30+2	END
31 through 37+2	Cancelled

Operating Instructions for the BRL-RSSR Program. As in Report ANL 6987, p. 73, with the following changes:

- No change
- No change
- Cancelled
- The first line is changed as follows:
A₁. Format (E 18, 10)
The remaining lines are not changed. (See Appendix B)

Example

The input and output of the BRLESC computations of the roots of the very badly conditioned polynomial

$$P(x) = 7056. x^6 - 51972. x^5 + 22559.36x^4 + 661360.64x^3 - 2101084.2x^2 + 2490368. x - 1048576. = 0$$

are given in Appendix C.

Availability. Copies of the punched card decks for POLYR and RSSR may be obtained from Systems Programming Branch, Computer Support Division.

BIBLIOGRAPHY

1. Bareiss, E.H. and Hamelink, R. , RSSR Routine, A Root-Squaring and Subresultant Procedure for Finding Zeros of Real Polynomials, Argonne National Laboratory - 6987, 1965.
2. Bareiss, E.H. , The Numerical Solution of Polynomial Equations and the Resultant Procedures , Mathematical Methods for Digital Computers, Vol. 2, 1967.

APPENDIX A

```

SUBROUTINE POLYR(N,COEFF,ROOTS,D)
DIMENSION A(51,3),IA(51,3),ROOTS(2,N),D(1),COEFF(1)
TYPE INTEGER DEGREE
DEGREE=N
N1=DEGREE+1
M=10
MMAX=15
DELTA=0.0001
EPSILON=0.000001
DO 14 I=1,N1
A(I,1)=COEFF(I)
IA(I,1)=0
CALL SCALE(A(I,1),IA(I,1))
14 CONTINUE
CALL RSSR(A,IA,ROOTS,DEGREE,M,MMAX,DELTA,EPSILON,D)
IF(N1-(DEGREE+1))29,29,26
26 RETURN
29 PRINT 30
30 FORMAT(21HOSOME ROOTS NOT FOUND)
RETURN
END

SUBROUTINE RSSR (A,IA,ROOTS,DEGREE,M,MMAX,DELTA,EPSILON,D)
DIMENSION A(51,3),IA(51,3),ROOTS(2,50),D(51),RCMOD(50),MRMOD(50),
1NONRT(50),MNONRT(50)
TYPE INTEGER DEGREE
N=DEGREE
IF (N) 1,1,3
1 DEGREE=NCUR
2 RETURN
3 N1=N+1
N2=N1+1
DO 5 I=1,N
K=N2-I
IF (A(K,1)) 6,4,6
4 J=N1-I
ROOTS(1,J)=0.0
ROOTS(2,J)=0.0
5 CONTINUE
DEGREE=0
GO TO 2
6 N1=K
N=K-1
NCUR=N
NL=N
7 CALL ROOTSQ (A,IA,NCUR,M)
CALL REALROOT (A,IA,M,NCUR,DELTA,EPSILON,RCMOD,MRMOD,
1NONRT,MNONRT,NCC,ROOTS)
IF (NCC) 12,12,9
8 N1=NCUR+1
CALL COMPROCT (A,IA,RCMOD,ROOTS,M,MNONRT,NONRT,
1MRMOD,NCC,DELTA,EPSILON,NCUR)
IF (NCUR) 12,12,9
9 IF (NL-NCUR) 11,11,10
10 NL=NCUR
GO TO 7
11 M=M+1
IF (MMAX-M) 1,7,7
12 CALL RECON (ROOTS,A(1,1),IA(1,1),D,DEGREE)
GO TO 1

```

END

```
SUBROUTINE ROOTSQ (A,IA,NCUR,MM)
DIMENSION A(51,3),IA(51,3)
N1=NCUR+1
DO 1 J=1,N1
  A(J,2)=A(J,1)
  IA(J,2)=IA(J,1)
  A(J,3)=0.0
  IA(J,3)=0
1 CONTINUE
DO 9 M=1,MM
  DO 6 J=1,N1
    K1=N1-J
    K2=J-1
    KM=XMINOF(K1,K2)
    IF(KM) 11,*,11
11 DO 4 L=1,KM
    LR=XMODF(L,2)
    JL=J-L
    JLP=J+L
    IF (LR) 2,2,3
  2 X=A(JL,2)*A(JLP,2)
    IX=IA(JL,2)+IA(JLP,2)
    CALL SCALF (X,IX)
    CALL ADD (A(J,3),IA(J,3),X,IX,A(J,3),IA(J,3))
    GO TO 4
  3 X=A(JL,2)*A(JLP,2)
    IX=IA(JL,2)+IA(JLP,2)
    CALL SCALE (X,IX)
    CALL SUBTRACT (A(J,3),IA(J,3),X,IX,A(J,3),IA(J,3))
  4 CONTINUE
    A(J,3)=2.0*A(J,3)
    CALL SCALF (A(J,3),IA(J,3))
    X=A(J,2)**2
    IX=IA(J,2)+IA(J,2)
    CALL SCALE (X,IX)
    CALL ADD (A(J,3),IA(J,3),X,IX,A(J,3),IA(J,3))
    JR=XMODF (J,2)
    IF (JR) 5,5,6
  5 A(J,3)=-A(J,3)
  6 CONTINUE
    IF (MM-M) 9,9,7
  7 DO 8 J=1,N1
    A(J,2)=A(J,3)
    IA(J,2)=IA(J,3)
    A(J,3)=0.0
    IA(J,3)=0
  8 CONTINUE
  9 CONTINUE
  RETURN
END
```

```
SUBROUTINE REALROOT (A,IA,M,NCUR,DELTA,EPSILON,ROMOD,MROMOD,
INONRT,MNONRT,NCO,ROOTS)
DIMENSION A(51,3),IA(51,3),ROOTS(2,50),ROMOD(50),MROMOD(50),
INONRT(50),MNONRT(50),RATIO(51),IPIV(51),ARED(50),IARED(50)
RATIO(1)=1.0
DO 4 I=2,NCUR
  I1=XMODF(I,2)
```

```

      IF (A(I,3)) 2,1,2
1  RATIO(I)=C.0
   GO TO 4
2  T=A(I,2)*A(I,2)
   IT=IA(I,2)+IA(I,2)
   CALL SCALL (T,IT)
   T=T/A(I,3)
   IT=IT-IA(I,3)
   IF (IT-2) 50,50,1
50  IF (IT+2) 1,51,51
51  CALL UNSCALE (T,IT)
   RATIO(I)=T
   IF (I1) 3,3,4
3  RATIO(I)=-RATIO(I)
4  CONTINUE
   RATIO(NCUR+1)=1.0
   IPIV(1)=1
   IPIV(NCUR+1)=1
   DO 7 I=2,NCUR
   X=ABS(RATIO(I)-1.0)
   IF (X-DELTA) 5,6,6
5  IPIV(I)=1
   GO TO 7
6  IPIV(I)=0
7  CONTINUE
   NCUR1=NCUR+1
   I1=0
   MULT=0
   I=1
   I4=1
8  I1=I1+1
   I2=I1+I
   MULT=MULT+1
   IF (IPIV(I2)) 8,8,9
9  ROMOD(I4)=A(I2,3)/A(I,3)
   IROMOD=IA(I2,3)-IA(I,3)
   CALL SCALE (ROMOD(I4),IROMOD)
   IF (ROMOD(I4)) 10,11,11
10 ROMOD(I4)=-ROMOD(I4)
11 CALL DOUBLOG (ROMOD(I4),IROMOD,XN,IXN)
   T=2 **M*I1
   XN=XN/T
   CALL SCALE (XN,IXN)
   CALL DOUBLEXP (XN,IXN,ROMOD(I4),IROMOD)
   IF (IROMOD-74 ) 100,100,101
100 IF (IROMOD+74 ) 101,101,102
101 ROMOD(I4)=0.0
   IROMOD=0
   GO TO 103
102 CALL UNSCALE (ROMOD(I4),IROMOD)
103 MROMOD(I4)=MULT
   IF (NCUR+1-I2) 13,13,12
12 I=I2
   I4=I4+1
   MULT=0
   I1=0
   GO TO 8
13 Q=C.0
   NCO=0
   DO 22 I=1,I4

```

```

KL=I4+1-I
W=-ROMOD(KL)
I5=MRCMOD(KL)
DO 20 J=1,I5
J=J
14 CALL TEST (A,IA,W,Q,NCUR,ROMOD(KL),EPSILON,K)
IF (K) 17,17,15
15 ROOTS(1,NCUR)=-W
ROOTS(2,NCUR)=0.0
ARED(1)=A(1,1)
IARED(1)=IA(1,1)
DO 16 L=2,NCUR
Y=ARED(L-1)*W
IY=IARED(L-1)
CALL SCALE (Y,IY)
CALL SUBTRACT (A(L,1),IA(L,1),Y,IY,ARED(L),IARED(L))
A(L,1)=ARED(L)
IA(L,1)=IARED(L)
16 CONTINUE
GO TO 19
17 IF (W) 18,21,21
18 W=-W
GO TO 14
19 NCUR=NCUR-1
20 CONTINUE
GO TO 22
21 NCC=NCO+1
NONRT(NCC)=KL
MNONRT(NCC)=I5+1-J
22 CONTINUE
RETURN
END

```

```

SUBROUTINE COMPROOT (A,IA,ROMOD,ROOTS,M,MNONRT,NONRT,
IMKCMOD,NCO,DELTA,EPSILON,NCUR)
DIMENSION A(51,3),IA(51,3),ROMOD(50),ROOTS(2,50),SR(51,3),ISR(51,3)
1),SROMOD(50),SROOTS(2,50),MNONRT(50),NONRT(50),MSROMOD(50),
2NSCNR(49),MSNR(49),MKROMOD(50),D(2),R(2),B(49)
DO 23 I=1,NCO
JA=NONRT(I)
I1=MNONRT(I)
I1=I1/2
IF (I1) 1,1,2
1 I1=1
2 IF (ROMOD(JA)) 3,23,3
3 Q=ROMOD(JA)
DO 22 J=1,I1
300 CALL SUBRES (A,IA,NCUR,SR,ISR,Q)
IF (NCUR-4) 4,100,5
100 NSCUR=2
GO TO 101
4 NSCUR=1
J2=1
GO TO 6
5 NSCUR=NCUR-3
101 J2=NSCUR
6 LL=NSCUR+1
IF (NSCUR-1) 7,7,9
7 IF (SR(1,1)) 8,10,8
8 X=SR(1,1) & IX=ISR(1,1)

```

```

Y=SR(2,1) & IY=ISR(2,1)
CALL UNSCALE (X,IX)
CALL UNSCALE (Y,IY)
SRCOTS(1,1)=-Y/X
NSCUR=0
GO TO 11
9 CALL ROOTSQ (SR,ISR,NSCUR,M)
CALL REALROOT (SR,ISR,M,NSCUR,DELTA,EPSILON,SROMOD,MSROMOD,
INSONRT,MSNORT,NSC0,SROOTS)
IF (J2-NSCUR) 10,10,11
10 SROOTS(1,J2)=0.0
11 SROOTS(1,J2)=SROOTS(1,J2)*ROMOD(JA)
T=ROMOD(JA)+ROMOD(JA)
IF (SROOTS(1,J2)-T) 12,16,15
12 W=SROOTS(1,J2)
WE=ROMOD(JA)*ROMOD(JA)
CALL TEST (A,IA,W,WE,NCUR,ROMOD(JA),EPSILON,K)
IF (K) 15,15,13
13 ROOTS(1,NCUR)=-W/2.0
T=4.0*WE
U=W*W
T=T-U
IF (T) 201,201,202
201 T=-T
U=DSQRT (T)
ROOTS(1,NCUR)=ROOTS(1,NCUR)-U/2.0
ROOTS(1,NCUR-1)=-(-W-U)/2.0
ROOTS(2,NCUR)=0.0
ROOTS(2,NCUR-1)=0.0
GO TO 204
202 U=DSQRT(T)
203 ROOTS(2,NCUR)=U/2.0
ROOTS(1,NCUR-1)=ROOTS(1,NCUR)
ROOTS(2,NCUR-1)=-ROOTS(2,NCUR)
204 D(1)=W
D(2)=WE
CALL QUADIV (NCUR,A,IA,R,D,B)
JX=NCUR-1
DO 14 JY=1,JX
A(JY,1)=R(JY)
IA(JY,1)=0
CALL SCALE (A(JY,1),IA(JY,1))
14 CONTINUE
NCUR=NCUR-2
GO TO 22
15 W=-W
CALL TEST (A,IA,W,WE,NCUR,ROMOD(JA),EPSILON,K)
IF (K) 16,16,13
16 IF (J2-(NSCUR+1)) 23,17,19
17 IF (J2-1) 23,23,18
18 J2=J2-1
SROOTS(1,J2)=0.0
GO TO 12
19 IF (SROOTS(1,J2)-SROOTS(1,J2-1)) 20,21,20
20 J2=J2-1
GO TO 11
21 J2=J2-1
GO TO 16
22 CONTINUE
23 CONTINUE

```

RETURN
END

```
SUBROUTINE TEST (A,IA,W,C,N,ROMOD,EPSILON,K)
DIMENSION A(51,3),IA(51,3),B(3),IB(3),T(2),E(2),C(51)
B(1)=0.0%IX=0%IW=0
IB(1)=0
R(2)=A(1,1)
IB(2)=IA(1,1)
DO 2 I=1,N
X=W*B(2)
IX=IB(2)
CALL SCALE (X,IX)
Y=Q*R(1)
IY=IB(1)
CALL SCALE (Y,IY)
CALL ADD (X,IX,Y,IY,Z,IZ)
CALL SUBTRACT (A(I+1,1),IA(I+1,1),Z,IZ,B(3),IB(3))
IF (N-I) 2,2,1
1 B(1)=R(2)
  IB(1)=IB(2)
  B(2)=R(3)
  IB(2)=IB(3)
2 CONTINUE
3 KOUNT=1
  CEPSIL=EPSILON
  T(1)=0.0%T(2)=0.0
  N1=N+1
  X=2.0*EPSILON
  Y=X*ROMOD
  E(1)=ROMOD+Y
  E(2)=ROMOD+CEPSIL*ROMOD
  DO 7 I=1,N1
    IF (A(I,1)) 4,5,5
  4 C(I)=-A(I,1)%IC=IA(I,1)
    GO TO 6
  5 C(I)=A(I,1)%IC=IA(I,1)
  6 CALL UNSCALE (C(I),IC)
    T(1)=T(1)*E(1)+C(I)
    T(2)=T(2)*E(2)+C(I)
  7 CONTINUE
    DIF=T(1)-T(2)
  8 IF (Q) 10,9,10
  9 IF (B(3)) 10,11,11
 10 B(3)=-B(3)
 11 IF (IB(3)-74 ) 100,100,12
 100 IF (IB(3)+74 ) 12,101,101
 101 CALL UNSCALE (B(3),IB(3))
    IF (DIF-B(3)) 12,12,17
 12 K=0
    IF (KOUNT-2) 13,16,16
 13 IF (Q) 15,14,15
 14 IF (W) 15,16,16
 15 SENSE LIGHT 2
    KOUNT=KOUNT+1
 16 RETURN
 17 K=1
    GO TO 16
 18 IF (IB(2)-74 ) 102,102,12
 102 IF (IB(2)+74 ) 12,103,103
```



```

103 CALL UNSCALE (B(2),IB(2))
    IF (IB(3)-74 ) 104,104,12
104 IF (IB(3)+74 ) 12,105,105
105 CALL UNSCALE (B(3),IB(3))
    X=Q*B(2)*B(2)
    Y=W*B(2)*B(3)
    Z=B(3)*B(3)
    Y=X-Y+Z
    IF (Y) 19,17,20
19 Y=-Y
20 DIF=DIF*DIF
    IF (DIF-Y) 12,17,17
END

```

```

SUBROUTINE SUBRES (A,IA,N,SR,ISR,ROMOD)
DIMENSION A(51,3),IA(51,3),SR(51,3),ISR(51,3),C(51),B(50,3)
N1=N+1
T=1.0
DO 1 I=1,N
    J=N1-I
    T=T*ROMOD
    C(J)=A(J,1)*T
    IC=IA(J,1)
    CALL UNSCALE (C(J),IC)
1 CONTINUE
    C(N1)=A(N1,1)
    IC=IA(N1,1)
    CALL UNSCALE (C(N1),IC)
    IF (N-2) 12,12,2
2 N2=N-2
    DO 3 I=1,N2
        B(I,1)=0.0
        B(I,2)=0.0
3 CONTINUE
    I=2
    B(1,2)=C(1)
4 B(1,3)=C(1)-C(1,1)
    DO 5 J=2,N2
        B(J,3)=-B(J-1,2)-B(J,1)
5 CONTINUE
    IF (N-(3+1)) 100,6,6
6 I=I+1
    DO 7 J=1,N2
        B(J,1)=B(J,2)
        B(J,2)=B(J,3)
7 CONTINUE
    GO TO 4
100 IF (N-4) 14,101,8
101 IF (N-(2+1)) 104,102,102
102 I=I+1
    DO 103 J=1,2
        B(J,1)=B(J,2)
        B(J,2)=B(J,3)
103 CONTINUE
    GO TO 4
104 B(3,3)=-B(2,2)
    SR(3,1)=-C(5)+B(1,3) & ISR(3,1)=0
    SR(2,1)=B(2,3) & ISR(2,1)=0
    SR(1,1)=B(3,3) & ISR(1,1)=0
    CALL SCALE (SR(1,1),ISR(1,1))

```

```

CALL SCALE (SR(2,1),ISR(2,1))
CALL SCALE (SR(3,1),ISR(3,1))
GO TO 11
8 SR(N2,1)=C(N)-R(1,3) & ISR(N2,1)=0
SR(N2-1,1)=-C(N1)-R(2,3) & ISR(N2-1,1)=0
CALL SCALE (SR(N2,1),ISR(N2,1))
CALL SCALE (SR(N2-1,1),ISR(N2-1,1))
IF (N2-2) 11,11,9
9 DO 10 J=3,N2
K=N2+1-J
SR(K,1)=-R(J,3) & ISR(K,1)=0
CALL SCALE (SR(K,1),ISR(K,1))
10 CONTINUE
11 RETURN
12 SR(1,1)=C(1) & ISR(1,1)=0
SR(2,1)=-C(2) & ISR(2,1)=0
13 CALL SCALE (SR(1,1),ISR(1,1))
CALL SCALE (SR(2,1),ISR(2,1))
GO TO 11
14 SR(1,1)=-C(4)
SR(2,1)=C(3)-C(1)
ISR(1,1)=0
ISR(2,1)=0
GO TO 13
END

```

```

SUBROUTINE RECON (ROOTS,A,IA,D,N)
DIMENSION ROOTS(2,50),D(51)
X=ASIX=IA
CALL UNSCALE (X,IX)
DO 1 I=1,N
D(I)=0.0
1 CONTINUE
D(N+1)=1.0
I=1
NL=N-1
2 IF (ROOTS(2,1)) 3,7,3
3 T=ROOTS(1,I)*ROOTS(1,I)
U=ROOTS(2,I)*ROOTS(2,I)
T=T+U
U=2.0*ROOTS(1,I)
DO 5 J=1,NL
IF (I+J-N) 5,4,4
4 D(J)=D(J+2)+T*D(J)
D'(J)=D(J)-U*D(J+1)
5 CONTINUE
D(N)=T*D(N)
D(N)=D(N)-U*D(N+1)
D(N+1)=T*D(N+1)
I=I+2
6 IF (N-I) 10,2,2
7 DO 9 J=1,N
IF (J+I-N) 9,9,8
8 D(J)=D(J+1)-D(J)*ROOTS(1,I)
9 CONTINUE
D(N+1)=-D(N+1)*ROOTS(1,I)
I=I+1
GO TO 6
10 NS=N+1
DO 11 II=1,NS

```

```

D(11)=D(11)*X
11 CONTINUE
RETURN
END

```

```

SUBROUTINE QUADIV (N,A,IA,R,D,B)
DIMENSION A(51,3),IA(51,3),R(2),D(2),B(4)
B(1)=A(1,1)*IA(1,1)
CALL UNSCALE (R(1),IB)
IF (N-2) 4,4,1
1 AA=A(2,1)*IA(2,1)
CALL UNSCALE (AA,IAA)
B(2)=AA-R(1)*D(1)
IF (N-3) 4,4,2
2 NT=N-1
DO 3 I=3,NT
XN=R(I-1)*D(1)
YN=R(I-2)*D(2)
AA=A(I,1)*IA(I,1)
CALL UNSCALE (AA,IAA)
B(I)=AA-(XN+YN)
3 CONTINUE
4 XN=R(N-1)*D(1)
YN=R(N-2)*D(2)
AA=A(N,1)*IA(N,1)
CALL UNSCALE (AA,IAA)
R(1)=AA-(XN+YN)
AA=A(N+1,1)*IA(N+1,1)
CALL UNSCALE (AA,IAA)
R(2)=AA-R(N-1)*D(2)
RETURN
END

```

```

SUBROUTINE DCURLOG (X,IX,Y,IY)
T=64.0
IF (X) 1,5,3
1 PRINT 2
2 FORMAT (46H( THE LOG OF A NON-POSITIVE NUMBER IS REQUESTED)
5 Y=0.0
IY=0
GO TO 4
3 TC=IX*
Y=ALOG(X)+T0*ALOG(T)
IY=0
CALL SCALE (Y,IY)
4 RETURN
END

```

```

SUBROUTINE DCURLEXP (X,IX,Z,IZ)
Z=EXP(X)*IZ=0
IF (IX) 1,9,6
1 I=-IX*11=6*1
DO 4 J=1,11
K=XMODF(IZ,2)
IF (K) 2,3,2
2 IZ=IZ-1
Z=64.0*Z
3 IZ=IZ/2
Z=DSORT(IZ)
CALL SCALE (Z,IZ)

```

```

4 CONTINUE
5 RETURN
6 I=6*IX
DO 7 J=1,I
Z=Z+Z*I Z=IZ+IZ
CALL SCALE (Z,IZ)
7 CONTINUE
GO TO 5
8 CALL SCALE (Z,IZ)
GO TO 5
END

```

```

SUBROUTINE ADD (X,IX,Y,IY,Z,IZ)
IF (X) 3,1,3
1 Z=Y
IZ=IY
2 RETURN
3 IF (Y) 5,4,5
4 Z=X
IZ=IX
GO TO 2
5 IDIFF=IX-IY
IF (IDIFF) 6,7,7
6 IA=IY
A=Y
B=X
IDIFF=-IDIFF
GO TO 8
7 IA=IX
A=X
B=Y
8 IF (16-IDIFF) 9,9,10
9 Z=A
IZ=IA
GO TO 2
10 IF (IDIFF) 11,12,11
11 DO 12 I=1,IDIFF
B=B/64.0
12 CONTINUE
Z=A+B
IZ=IA
CALL SCALE (Z,IZ)
GO TO 2
END

```

```

SUBROUTINE SUBTRACT (X,IX,Y,IY,Z,IZ)
W=-Y
CALL ADD (X,IX,W,IY,Z,IZ)
RETURN
END

```

```

SUBROUTINE SCALE (X,IX)
TYPE DOUBLE X,Y
REC64=1.0/64.0
IF (IX) 1,11,2
1 Y=-X
GO TO 3
2 Y=X
3 IF (64.0-Y) 4,5,5
4 Y=Y/64.0

```

```

IX=IX+1
GO TO 3
5 IF (Y-REC64) 6,7,7
6 Y=Y*64.0
IX=IX-1
GO TO 5
7 IF (X) 8,9,9
8 X=-Y
GO TO 10
9 X=Y
10 RETURN
11 IX=0
GO TO 10
END

```

```

SUBROUTINE UNSCALE (X,IX)
IF (IX+64) 3,4,4
3 X=0.0
IX=0
GO TO 2
4 IF (IX-84) 5,5,6
6 X=1.0E+153
IX=0
PRINT 7
7 FORMAT (25HDEXP. OVERFLOW IN UNSCALE)
GO TO 2
5 IF (IX) 1,2,1
1 X=X*64.0**IX
8 IX=0
2 RETURN
END

```

F

APPENDIX B

```

* C-907 FINDING ZEROS OF REAL POLYNOMIALS
C MAIN PROGRAM
C DOES INPUT AND OUTPUT, AND CALLS RSSR
  DIMENSION A(51,3),IA(51,3),ROOTS(2,50),TITLE(10),D(51)
  TYPE INTEGER DEGREE
1 READ 2, TITLE
2 FORMAT (10A9)
4 PRINT 2, TITLE
5 FORMAT (1H1,10A9)
  READ 6, DEGREE,M,MMAX,DELTA,EPSILON
6 FORMAT (3I6,2E12.4)
  IF (M) 7,7,8
7 M=10
  MMAX=15
  DELTA=0.0001
  EPSILON=0.000001
8 PRINT 9, DEGREE,M,MMAX,DELTA,EPSILON
9 FORMAT (3HDEGREE=13,17H NO. OF ROOT SQ=13,7H MMAX=13,7
17H DELTA=E12.3,10H EPSILON=E12.3)
  N1=DEGREE+1
  PRINT 10
10 FORMAT (19HINPUT COEFFICIENTS)
11 DO 14 I=1,N1
  READ 12, X
12 FORMAT (E18.10)
  A(I,1)=X
  IA(I,1)=0
  J=I-1
  PRINT 13, J,X
13 FORMAT (3HCA(12,3H)= 1PE18.10)
  CALL SCALE (A(I,1),IA(I,1))
14 CONTINUE
19 N2=DEGREE
  CALL RSSR (A,IA,ROOTS,DEGREE,M,MMAX,DELTA,EPSILON,D)
20 IF (N1-(DEGREE+1)) 29,29,21
21 PRINT 22
22 FORMAT (24HROOTS OF THE POLYNOMIAL)
  PRINT 23
23 FORMAT (42HREAL PART IMAGINARY PART)
  K=DEGREE+1
  DO 25 I=K,N2
  X=ROOTS(1,I)
  Y=ROOTS(2,I)
  PRINT 24,X,Y
24 FORMAT (1H01PE24.15,1PE26.15)
25 CONTINUE
  IF (DEGREE) 26,26,29
26 PRINT 27
27 FORMAT (25HRECONSTRUCTED POLYNOMIAL)
  DO 28 I=1,N1
  X=D(I)
  J=I-1
  PRINT 13, J,X
28 CONTINUE
  GO TO 1
29 PRINT 30
30 FORMAT (21HSOME ROOTS NOT FOUND)
  GO TO 1
  END
  SUBROUTINE RSSR (A,IA,ROOTS,DEGREE,M,MMAX,DELTA,EPSILON,D)

```

```

    DIMENSION A(51,3),IA(51,3),ROOTS(2,50),D(51),RCMOD(50),MRCMOD(50),
    INONRT(50),MNONRT(50)
    TYPE INTEGER DEGREE
    N=DEGREE
    IF (N) 1,1,3
1  DEGREE=NCUR
2  RETURN
3  N1=N+1
  N2=N1+1
  DO 5 I=1,N
    K=N2-I
    IF (A(K,1)) 5,4,6
4  J=N1-I
  ROOTS(1,J)=0.0
  ROOTS(2,J)=0.0
5  CONTINUE
  DEGREE=C
  GO TO 2
6  N1=K
  N=K-1
  NCUR=N
  NL=N
7  CALL ROOTSQ (A,IA,NCUR,M)
  CALL REALROOT (A,IA,M,NCUR,DELTA,EPSILON,RCMOD,"RCMOD",
  INONRT,MNONRT,NCO,ROOTS)
  IF (NCO) 12,12,8
8  N1=NCUR+1
  CALL COMPROOT (A,IA,RCMOD,ROOTS,M,MNONRT,INONRT,
  IMRCMOD,NCO,DELTA,EPSILON,NCUR)
  IF (NCUR) 12,12,9
9  IF (NL-NCUR) 11,11,10
10 NL=NCUR
  GO TO 7
11 M=M+1
  IF (MMAX-M) 1,7,7
12 CALL RECON (ROOTS,A(1,1),IA(1,1),D,DEGREE)
  GO TO 1
END

```

```

SUBROUTINE ROOTSQ (A,IA,NCUR,MM)
DIMENSION A(51,3),IA(51,3)
N1=NCUR+1
DO 1 J=1,N1
  A(J,2)=A(J,1)
  IA(J,2)=IA(J,1)
  A(J,3)=0.0
  IA(J,3)=0
1 CONTINUE
DO 9 M=1,MM
DO 6 J=1,N1
  K1=N1-J
  K2=J-1
  KM=XMINOF(K1,K2)
  IF (KM) 11,4,11
11 DO 4 L=1,KM
  LR=XMODF(L,2)
  JL=J-L
  JLP=J+L
  IF (LR) 2,2,3
2 X=A(JL,2)*A(JLP,2)

```

```

IX=IA(JL,2)+IA(JLP,2)
CALL SCALE (X,IX)
CALL ADD (A(J,3),IA(J,3),X,IX,A(J,3),IA(J,3))
GO TO 4
3 X=A(JL,2)*A(JLP,2)
IX=IA(JL,2)+IA(JLP,2)
CALL SCALE (X,IX)
CALL SUBTRACT (A(J,3),IA(J,3),X,IX,A(J,3),IA(J,3))
4 CONTINUE
A(J,3)=2.0*A(J,3)
CALL SCALE (A(J,3),IA(J,3))
X=A(J,2)**2
IX=IA(J,2)+IA(J,2)
CALL SCALE (X,IX)
CALL ADD (A(J,3),IA(J,3),X,IX,A(J,3),IA(J,3))
JR=X*CDF (J,2)
IF (JR) 5,5,6
5 A(J,3)=-A(J,3)
6 CONTINUE
IF (YK-M) 9,9,7
7 DO 8 J=1,N1
A(J,2)=A(J,3)
IA(J,2)=IA(J,3)
A(J,3)=0.0
IA(J,3)=0
8 CONTINUE
9 CONTINUE
RETURN
END

```

```

SUBROUTINE REALROOT (A,IA,M,NCUR,DELTA,EPSILON,ROMOD,MROMOD,
1 INCNRT,MNCNRT,NCO,ROOTS)
DIMENSION A(51,3),IA(51,3),ROOTS(2,50),ROMOD(50),MROMOD(50),
1 INCNRT(50),MNCNRT(50),RATIO(51),IPIV(51),ARED(50),IARED(50)
RATIO(1)=1.0
DO 4 I=2,NCUR
I1=XCDF(I,2)
IF (A(I,3)) 2,1,2
1 RATIO(I)=0.0
GO TO 4
2 T=A(I,2)*A(I,2)
IT=IA(I,2)+IA(I,2)
CALL SCALE (T,IT)
T=T/A(I,3)
IT=IT-IA(I,3)
IF (IT-2) 50,50,1
50 IF (IT+2) 1,51,51
51 CALL UNSCALE (T,IT)
RATIO(I)=T
IF (I1) 3,3,4
3 RATIO(I)=-RATIO(I)
4 CONTINUE
RATIO(NCUR+1)=1.0
IPIV(1)=1
IPIV(NCUR+1)=1
DO 7 I=2,NCUR
X=ABSF(RATIO(I)-1.0)
IF (X-DELTA) 5,6,6
5 IPIV(I)=1
GO TO 7

```



```

6 IPIV(I)=C
7 CONTINUE
  NCUR=NCUR+1
  I1=0
  MULT=0
  I=1
  I4=1
8 I1=I1+1
  I2=I1+I
  MULT=MULT+1
  IF (IPIV(I2)) 8,8,9
9 ROMOD(I4)=A(I2,3)/A(I,3)
  IROMOD=IA(I2,3)-IA(I,3)
  CALL SCALE (ROMOD(I4),IROMOD)
  IF (ROMOD(I4)) 10,11,11
10 ROMOD(I4)=-ROMOD(I4)
11 CALL DQURLOG (ROMOD(I4),IROMOD,XN,IXN)
  T=2 **M*I1
  XN=XN/T
  CALL SCALE (XN,IXN)
  CALL DQURLEXP (XN,IXN,ROMOD(I4),IROMOD)
  IF (IROMOD-74 ) 100,100,101
100 IF (IROMOD+74 ) 101,101,102
101 ROMOD(I4)=0.0
  IROMOD=C
  GO TO 103
102 CALL UNSCALE (ROMOD(I4),IROMOD)
103 MROMOD(I4)=MULT
  IF (NCUR+1-12) 13,13,12
12 I=I2
  I4=I4+1
  MULT=C
  I1=0
  GO TO 8
13 Q=0.0
  NCC=C
  DO 22 I=1,I4
    KL=I4+1-I
    W=-ROMOD(KL)
    I5=MROMOD(KL)
    DO 20 J=1,I5
      J=J
14 CALL TEST (A,IA,W,Q,NCUR,ROMOD(KL),EPSILON,K)
    IF (K) 17,17,15
15 ROOTS(1,NCUR)=-W
    ROOTS(2,NCUR)=0.0
    ARED(1)=A(1,1)
    IARED(1)=IA(1,1)
    DO 16 L=2,NCUR
      Y=ARED(L-1)*W
      IY=IARED(L-1)
      CALL SCALE (Y,IY)
      CALL SUBTRACT (A(L,1),IA(L,1),Y,IY,ARED(L),IARED(L))
      A(L,1)=ARED(L)
      IA(L,1)=IARED(L)
16 CONTINUE
    GO TO 19
17 IF (W) 18,21,21
18 W=-W
    GO TO 14

```

```

19 NCUR=NCUR-1
20 CONTINUE
   GO TO 22
21 NCC=NCO+1
   NONRT(NCO)=KL
   MNONRT(NCC)=I5+1-J
22 CONTINUE
   RETURN
   END

SUBROUTINE COMPROOT (A,IA,ROMOD,ROOTS,M,MNONPT,NONRT,
1MROMOD,NCO,DELTA,EPSILON,NCUR)
  DIMENSION A(51,3),IA(51,3),ROMOD(50),ROOTS(2,50),SR(51,3),ISR(51,3
1),SROMOD(50),SROOTS(2,50),MNONRT(50),NONRT(50),MSROMOD(50),
2NSCNRT(49),MSNORT(49),MROMOD(50),D(2),K(2),R(49)
  DO 23 I=1,NCO
    JA=NONRT(I)
    I1=MNONRT(I)
    I1=I1/2
    IF (I1) 1,1,2
  1 I1=1
  2 IF (ROMOD(JA)) 3,23,3
  3 Q=ROMOD(JA)
  DO 22 J=1,I1
300 CALL SUBRES (A,IA,NCUR,SR,ISR,Q)
    IF (NCUR-4) 4,100,5
100 NSCUR=2
    GO TO 101
  4 NSCUR=1
    J2=1
    GO TO 6
  5 NSCUR=NCUR-3
101 J2=NSCUR
  6 LL=NSCUR+1
    IF (NSCUR-1) 7,7,7
  7 IF (SR(1,1)) 8,10,8
  8 X=SR(1,1) & IX=ISR(1,1)
    Y=SR(2,1) & IY=ISR(2,1)
    CALL UNSCALE (X,IX)
    CALL UNSCALE (Y,IY)
    SROOTS(1,1)=-Y/X
    NSCUR=0
    GO TO 11
  9 CALL ROOTSQ (SR,ISR,NSCUR,M)
    CALL REALROOT (SR,ISR,M,NSCUR,DELTA,EPSILON,SROMOD,MSROMOD,
1NSCNRT,MSNORT,NSCO,SROOTS)
    IF (J2-NSCUR) 10,10,11
10 SROOTS(1,J2)=0.0
11 SROOTS(1,J2)=SROOTS(1,J2)*ROMOD(JA)
    T=ROMOD(JA)+ROMOD(JA)
    IF (SROOTS(1,J2)-T) 12,16,16
12 W=SROOTS(1,J2)
    WE=ROMOD(JA)*ROMOD(JA)
    CALL TEST (A,IA,W,WE,NCUR,ROMOD(JA),EPSILON,K)
    IF (K) 15,15,13
13 ROOTS(1,NCUR)=-W/2.0
    T=4.0*WE
    U=W*W
    T=T-U
    IF (T) 201,201,202

```

```

201 T=-T
    U=DSQRT (T)
    ROOTS(1,NCUR)=ROOTS(1,NCUR)-U/2.0
    ROOTS(1,NCUR-1)=- (W-U)/2.0
    ROOTS(2,NCUR)=0.0
    ROOTS(2,NCUR-1)=0.0
    GO TO 204
202 U=DSQRT(T)
203 ROOTS(2,NCUR)=U/2.0
    ROOTS(1,NCUR-1)=ROOTS(1,NCUR)
    ROOTS(2,NCUR-1)=-ROOTS(2,NCUR)
204 D(1)=W
    D(2)=WE
    CALL QUADIV (NCUR,A,IA,R,D,B)
    JX=NCUR-1
    DO 14 JY=1,JX
        A(JY,1)=B(JY)
        IA(JY,1)=0
        CALL SCALE (A(JY,1),IA(JY,1))
14 CONTINUE
    NCUR=NCUR-2
    GO TO 22
15 W=-W
    CALL TEST (A,IA,W,WE,NCUR,ROMOD(JA),EPSILON,K)
    IF (K) 15,16,13
16 IF (J2-(NSCUR+1)) 23,17,19
17 IF (J2-1) 23,23,13
18 J2=J2-1
    SROOTS(1,J2)=0.0
    GO TO 12
19 IF (SROOTS(1,J2)-SROOTS(1,J2-1)) 20,21,20
20 J2=J2-1
    GO TO 11
21 J2=J2-1
    GO TO 16
22 CONTINUE
23 CONTINUE
    RETURN
    END

SUBROUTINE TEST (A,IA,W,Q,N,ROMOD,EPSILON,K)
    DIMENSION A(51,3),IA(51,3),B(3),IR(3),T(2),E(2),C(51)
    B(1)=0.0;IX=0;IW=0
    IB(1)=0
    R(2)=A(1,1)
    IR(2)=IA(1,1)
    DO 2 I=1,N
        X=W*B(2)
        IX=IR(2)
        CALL SCALE (X,IX)
        Y=Q*R(1)
        IY=IR(1)
        CALL SCALE (Y,IY)
        CALL ADD (X,IX,Y,IY,Z,IZ)
        CALL SUBTRACT (A(I+1,1),IA(I+1,1),Z,IZ,R(3),IR(3))
        IF (N-I) 2,2,1
1    B(1)=B(2)
        IB(1)=IB(2)
        B(2)=B(3)
        IB(2)=IB(3)

```

```

2 CONTINUE
3 KOUNT=1
  CEPSIL=EPSILON
  T(1)=C.0*T(2)=0.0
  N1=N+1
  X=2.0*EPSILON
  Y=X*ROMOD
  E(1)=ROMOD+Y
  E(2)=ROMOD+CEPSIL*ROMOD
  DO 7 I=1,N1
    IF (A(I,1)) 4,5,5
4 C(I)=-A(I,1)*IC=IA(I,1)
  GO TO 6
5 C(I)=A(I,1)*IC=IA(I,1)
6 CALL UNSCALE (C(I),IC)
  T(1)=T(1)*E(1)+C(I)
  T(2)=T(2)*E(2)+C(I)
7 CONTINUE
  DIF=T(1)-T(2)
8 IF (Q) 18,9,18
9 IF (P(3)) 10,11,11
10 R(3)=-R(3)
11 IF (IR(3)-74 ) 100,100,12
100 IF (IP(3)+74 ) 12,101,101
101 CALL UNSCALE (R(3),IB(3))
  IF (DIF-R(3)) 12,12,17
12 K=0
  IF (KOUNT-2) 13,16,16
13 IF (Q) 15,14,15
14 IF (W) 15,16,16
15 SENSE LIGHT 2
  KOUNT=KOUNT+1
16 RETURN
17 K=1
  GO TO 16
18 IF (IP(2)-74 ) 102,102,12
102 IF (IB(2)+74 ) 12,103,103
103 CALL UNSCALE (R(2),IB(2))
  IF (IR(3)-74 ) 104,104,12
104 IF (IP(3)+74 ) 12,105,105
105 CALL UNSCALE (R(3),IB(3))
  X=Q*B(2)*R(2)
  Y=W*P(2)*B(3)
  Z=R(3)*B(3)
  Y=X-Y+Z
  IF (Y) 19,17,20
19 Y=-Y
20 DIF=DIF*DIF
  IF (DIF-Y) 12,17,17
  END

SUBROUTINE SUBRES (A,IA,N,SR,ISR,ROMOD)
  DIMENSION A(51,3),IA(51,3),SR(51,3),ISR(51,3),C(51),B(50,3)
  N1=N+1
  T=1.0
  DO 1 I=1,N
    J=N1-I
    T=T*ROMOD
    C(J)=A(J,1)*T
    IC=IA(J,1)

```

```

      CALL UNSCALE (C(J),IC)
1  CONTINUE
      C(N1)=A(N1,1)
      IC=IA(N1,1)
      CALL UNSCALE (C(N1),IC)
      IF (N-2) 12,12,2
2  N2=N-2
      DO 3 I=1,N2
        B(I,1)=C,0
        B(I,2)=C,0
3  CONTINUE
      I=2
      B(1,2)=C(1)
4  B(1,3)=C(1)-B(1,1)
      DO 5 J=2,N2
        B(J,3)=-B(J-1,2)-B(J,1)
5  CONTINUE
      IF (N-(3+1)) 100,6,6
6  I=I+1
      DO 7 J=1,N2
        B(J,1)=B(J,2)
        B(J,2)=B(J,3)
7  CONTINUE
      GO TO 4
100 IF (N-4) 14,101,9
101 IF (N-(2+1)) 104,102,102
102 I=I+1
      DO 103 J=1,2
        B(J,1)=B(J,2)
        B(J,2)=B(J,3)
103 CONTINUE
      GO TO 4
104 B(3,3)=-B(2,2)
      SR(3,1)=-C(5)+B(1,3) & ISR(3,1)=0
      SR(2,1)=B(2,3) & ISR(2,1)=0
      SR(1,1)=B(3,3) & ISR(1,1)=0
      CALL SCALE (SR(1,1),ISR(1,1))
      CALL SCALE (SR(2,1),ISR(2,1))
      CALL SCALE (SR(3,1),ISR(3,1))
      GO TO 11
8  SR(N2,1)=C(N1)-B(1,3) & ISR(N2,1)=0
      SR(N2-1,1)=-C(N1)-B(2,3) & ISR(N2-1,1)=0
      CALL SCALE (SR(N2,1),ISR(N2,1))
      CALL SCALE (SR(N2-1,1),ISR(N2-1,1))
      IF (N2-2) 11,11,9
9  DO 10 J=3,N2
      K=N2-1-J
      SR(K,1)=-B(J,3) & ISR(K,1)=0
      CALL SCALE (SR(K,1),ISR(K,1))
10 CONTINUE
11 RETURN
12 SR(1,1)=C(1) & ISR(1,1)=0
      SR(2,1)=-C(2) & ISR(2,1)=0
13 CALL SCALE (SR(1,1),ISR(1,1))
      CALL SCALE (SR(2,1),ISR(2,1))
      GO TO 11
14 SR(1,1)=-C(4)
      SR(2,1)=C(3)-C(1)
      ISR(1,1)=0
      ISR(2,1)=0

```

GO TO 13
END

```

SUBROUTINE RECON (ROOTS,A,IA,D,N)
DIMENSION ROOTS(2,50),D(51)
X=A$IX=IA
CALL UNSCALE (X,IX)
DO 1 I=1,N
D(I)=0.0
1 CONTINUE
D(N+1)=1.0
I=1
NL=N-1
2 IF (ROOTS(2,I)) 3,7,3
3 T=ROOTS(1,I)*ROOTS(1,I)
U=ROOTS(2,I)*ROOTS(2,I)
T=T+U
U=2.0*ROOTS(1,I)
DO 5 J=1,NL
IF (I+J-N) 5,4,4
4 D(J)=D(J+1)+T*D(J)
D(J)=D(J)-U*D(J+1)
5 CONTINUE
D(N)=T*D(N)
D(N)=D(N)-U*D(N+1)
D(N+1)=T*D(N+1)
I=I+2
6 IF (N-I) 10,2,2
7 DO 9 J=1,N
IF (J+I-N) 9,4,4
8 D(J)=D(J+1)-D(J)*ROOTS(1,I)
9 CONTINUE
D(N+1)=-D(N+1)*ROOTS(1,I)
I=I+1
GO TO 6
10 NS=N+1
DO 11 II=1,NS
D(II)=D(II)*X
11 CONTINUE
RETURN
END

```

```

SUBROUTINE QUADIV (N,A,IA,R,D,B)
DIMENSION A(51,3),IA(51,3),R(2),D(2),B(49)
R(1)=A(1,1)$IB=IA(1,1)
CALL UNSCALE (R(1),IB)
IF (N-2) 4,4,2
1 AA=A(2,1)$IAA=IA(2,1)
CALL UNSCALE (AA,IAA)
B(2)=AA-R(2)*D(1)
IF (N-3) 4,4,2
2 NT=N-1
DO 3 I=3,NT
XN=B(I-1)*D(1)
YN=B(I-2)*D(2)
AA=A(I,1)$IAA=IA(I,1)
CALL UNSCALE (AA,IAA)
B(I)=AA-(XN+YN)
3 CONTINUE
4 XN=B(N-1)*D(1)

```

```

YN=R(N-2)*D(2)
AA=A(N,1)*IAA=IA(N,1)
CALL UNSCALE (AA,IAA)
R(1)=AA-(XN+YN)
AA=A(N+1,1)*IAA=IA(N+1,1)
CALL UNSCALE (AA,IAA)
R(2)=AA-R(N-1)*D(2)
RETURN
END

```

```

SUBROUTINE DCURLOG (X,IX,Y,IY)
T=64.0
IF (X) 1,5,3
1 PRINT 2
2 FORMAT (46HCTHE LOG OF A NON-POSITIVE NUMBER IS REQUESTED)
5 Y=C.0
IY=C
GO TO 4
3 TO=IX*
Y=ALOG(X)+TO*ALOG(T)
IY=C
CALL SCALE (Y,IY)
4 RETURN
END

```

```

SUBROUTINE DCURLEXP (X,IX,Z,IZ)
Z=EXP(X)*IZ=C
IF (IX) 1,6,6
1 I=-IX*I=6*I
DO 4 J=1,I
K=XMODF(IZ,2)
IF (K) 2,3,2
2 IZ=IZ-1
Z=64.0*I
3 IZ=IZ/2
Z=DSQRT(Z)
CALL SCALE (Z,IZ)
4 CONTINUE
5 RETURN
6 I=6*I
DO 7 J=1,I
Z=Z*Z*IZ=IZ*IZ
CALL SCALE (Z,IZ)
7 CONTINUE
GO TO 5
8 CALL SCALE (Z,IZ)
GO TO 5
END

```

```

SUBROUTINE ADD (X,IX,Y,IY,Z,IZ)
IF (X) 3,1,3
1 Z=Y
IZ=IY
2 RETURN
3 IF (Y) 5,4,5
4 Z=X
IZ=IX
GO TO 2
5 IDIFF=IX-IY
IF (IDIFF) 6,7,7

```

```

6  IA=IY
   A=Y
   R=X
   IDIFF=IDIFF
   GO TO 8
7  IA=IX
   A=X
   R=Y
8  IF (16-IDIFF) 9,9,10
9  Z=A
   IZ=IA
   GO TO 2
10 IF(IDIFF) 11,12,11
11 DO 12 I=1,IDIFF
   R=R/64.0
12 CONTINUE
   Z=A+R
   IZ=IA
   CALL SCALE (Z,IZ)
   GO TO 2
END

```

```

SUBROUTINE SUBTRACT (X,IX,Y,IY,Z,IZ)
W=-Y
CALL ADD (X,IX,W,IY,Z,IZ)
RETURN
END

```

```

SUBROUTINE SCALE (X,IX)
TYPE DOUBLE X,Y
R=64=1.0/64.0
IF (X) 1,11,2
1  Y=-X
   GO TO 3
2  Y=X
3  IF (64.0-Y) 4,5,5
4  Y=Y/64.0
   IX=IX+1
   GO TO 3
5  IF (Y-R*64) 6,7,7
6  Y=Y*64.0
   IX=IX-1
   GO TO 5
7  IF (X) 8,9,9
8  X=-Y
   GO TO 10
9  X=Y
10 RETURN
11 IX=0
   GO TO 10
END

```

```

SUBROUTINE UNSCALE (X,IX)
IF (IX+84) 3,4,4
3  X=C.0
   IX=0
   GO TO 2
4  IF (IX-84) 5,5,5
6  X=1.0F+153
   IX=0

```



```
PRINT 7  
7 FORMAT (25HEXP. OVERFLOW IN UNSCALE)  
GO TO 2  
5 IF (IX) 1,2,1  
1 X=X*64.0**IX  
8 IX=0  
2 RETURN  
END
```

E

APPENDIX C

INPUT

EXAMPLE 1

6 20 20 1.000E-4 1.000E-6
 7056.
 -51072.
 22559.36
 661360.64
 -2101084.2
 2490368.
 -1048576.

OUTPUT

EXAMPLE 1

DEGREE= 6 NO. OF ROOT SQ= 20 MMAX= 20
 DELTA= 0.100E-03 EPSILON= 0.100E-05

INPUT COEFFICIENTS

A(0)= 7.0560000000E 03
 A(1)= -5.1072000000E 04
 A(2)= 2.2559360000E 04
 A(3)= 6.6136064000E 05
 A(4)= -2.1010842000E 06
 A(5)= 2.4903680000E 06
 A(6)= -1.0485760000E 06

ROOTS OF THE POLYNOMIAL

REAL PART

IMAGINARY PART

-3.999687510094593E 00	0.000000000000000E 00
3.998749957314208E 00	0.000000000000000E 00
2.285716169209813E 00	0.000000000000000E 00
2.285716169209813E 00	0.000000000000000E 00
1.357365320213065E 00	0.000000000000000E 00
1.310236461141304E 00	0.000000000000000E 00

OUTPUT

RECONSTRUCTED POLYNOMIAL

A(0) = 7.0560700000E 03

A(1) = -5.1072009377E 04

A(2) = 2.2559385004E 04

A(3) = 6.6136077332E 05

A(4) = -2.1010846001E 06

A(5) = 2.4903682667E 06

A(6) = -1.0485760000E 06

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13. ABSTRACT This report describes the FORTRAN Subroutine POLYR and a related complete program BRL-RSSR for finding all roots (real and complex) of a real polynomial equation $P(x) = \sum_{i=0}^N A_i x^{N-i} = 0.$ <p>The method which is used combines the root squaring and the subresultant (extracting quadratic factors) procedures for calculating all roots, even multiple roots, of real polynomials. Reconstruction of the coefficients from the product</p> $\prod_{i=1}^N (x - \text{Root}_i),$ <p>which is included, serves as a means of checking.</p> <p>The described subroutine and program were adapted from a similar program in double precision described in Report ANL 6987, Argonne National Laboratory by Erwin H. Bareiss and Ronald Hamelink.</p>		

DD FORM 1473

REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

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